ECI750 Multimedia Data Compression

Lectures 13-14

Mathematical Preliminaries of Lossy Compression

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Lossy Compression

Overview

- In lossless compression schemes, rate is the general concern.
- In lossy compression schemes, the loss of information (distortion) associated with such schemes is also a concern.
- We will look at different ways of assessing the impact of the loss of information.
- We will also look at the rate-distortion theory.
- We will also look at some of the models used in the development of lossy compression schemes.

Lossy Compression

Introduction

- With lossless compression, only a limited amount of compression can be achieved.
- There is a floor, defined by the entropy of the source, below which we cannot drive the size of the compressed sequence.
- For lossless compression, entropy, like the speed of light, is a fundamental limit.
- So why did we study lossless compression, at all, then?
  - The storage or transmission resources available to us might be sufficient to handle our data requirements.
  - The consequences of a loss of information may be much more expensive than the cost of additional storage/transmission resources.
    - E.g., archiving of bank records: an error in the records could turn out to be much more expensive than the cost of buying additional storage.

Lossy Compression

Introduction (2)

- We can improve the amount of compression by accepting a certain degree of loss during compression.
- Performance measures are necessary to determine the efficiency of a lossy compression scheme.
- Using only the rate as a measure of performance is not sufficient anymore. Otherwise, the best lossy compression scheme would be to throw away all the data.
- Hence, an additional performance measure is needed such as a measure of the difference between the original and the reconstructed data (distortion).

Lossy Compression

Introduction (3)

- When we transmit no information, the rate is zero and the distortion is maximum.
- When we transmit all information, the distortion is zero and the rate is maximum.
- The study of the situations between these two extremes is called the *rate-distortion theory*.

Lossy Compression

Distortion Criteria

- The difference between x and x' is the distortion.
- Distortion can be judged by humans or machines
- The former class of methods is called subjective criteria.
- The latter class of methods is called objective criteria.

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Distortion Criteria (2)

- Subjective Criteria
  - Mean Opinion Score

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<u>Distortion Criteria (2)</u>

- Objective Criteria
  - Squared error measure

$$d(x, y) = (x - y)^2$$

  - Absolute difference measure

$$d(x, y) = |x - y|$$

  - Mean Squared Error (MSE)

$$\sigma^2 = \frac{1}{N}\sum_{n=1}^{N}(x_n - y_n)^2$$

  - Mean Absolute Differences (MAD)

$$d = \frac{1}{N}\sum_{n=1}^{N}|x_n - y_n|$$

  - Max Error Measure

$$d = \max_n |x_n - y_n|$$

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Distortion Criteria (3)

- Objective Criteria (2)
  - Relative Error Measures
    - SNR

$$SNR = \frac{\sigma_x^2}{\sigma_d^2}$$

$$SNR(dB) = 10log_{10}\frac{\sigma_x^2}{\sigma_d^2}$$

    - PSNR

$$PSNR(dB) = 10log_{10}\frac{x_{peak}^2}{\sigma_d^2}$$

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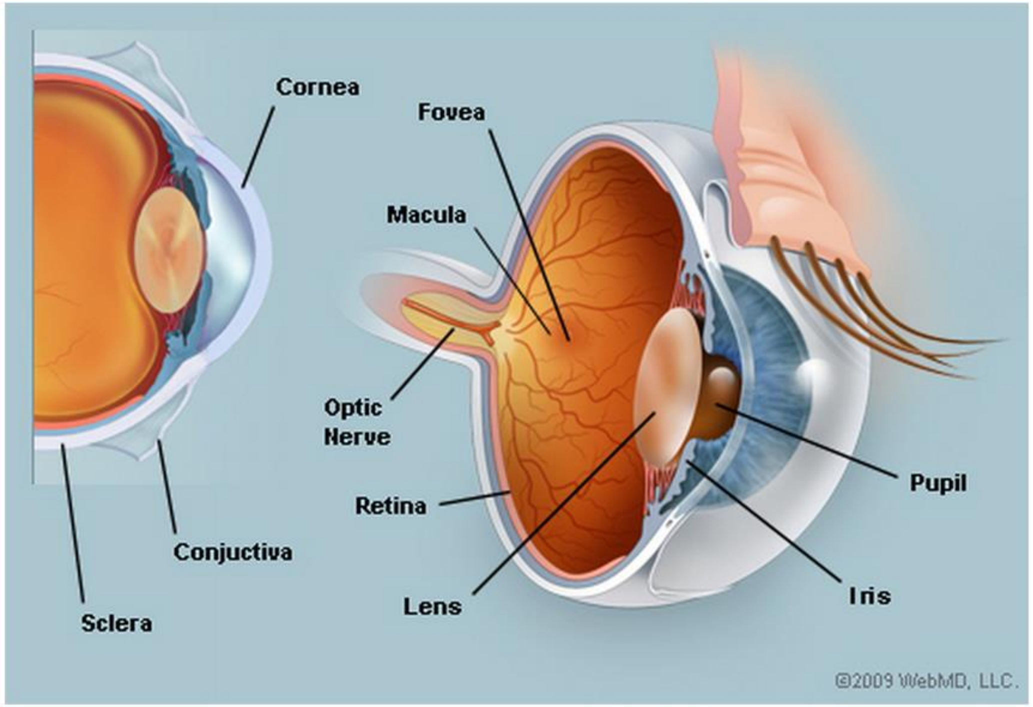
Human Visual System (HVS)

- The Human Eye
  - Globe-shaped with a lens in the front that focuses objects onto the retina in the back of the eye
  - Retina contains two types of receptors
    - Rods:
      - more sensitive to light than cones
      - in low-light most of our vision is due to rods
    - Cones:
      - Three types, each is most sensitive at different wavelengths of the visible spectrum.
      - The peak sensitivities of the cones are in the red, green, and blue regions of the visible spectrum.
      - Concentrated in a very small area of retina (fovea).



Lossy Compression

<u>Human Visual System (HVS) (2)</u>

- Rods are more in number, than the cones
- Cones are more closely packed in the fovea and provide better resolution.
- The muscles of the eye move the eyeball, positioning the image of the object on the fovea.
  - This becomes a drawback in low-light.
  - One way to improve what you see in low-light is to focus on one side of the object. This way, the object is imaged on the rods, which are more sensitive to light.

Lossy Compression

<u>Human Visual System (HVS) (3)</u>

- At a given instant, we cannot perceive the entire range of brightness.
- The eye adapts to an average brightness level.
- Just Noticeable Difference (JND)
  - If the background intensity is I, the centre intensity is $I + \Delta I$, JND is the minimal ΔI which makes the central object visible.

contrast sensitivity test

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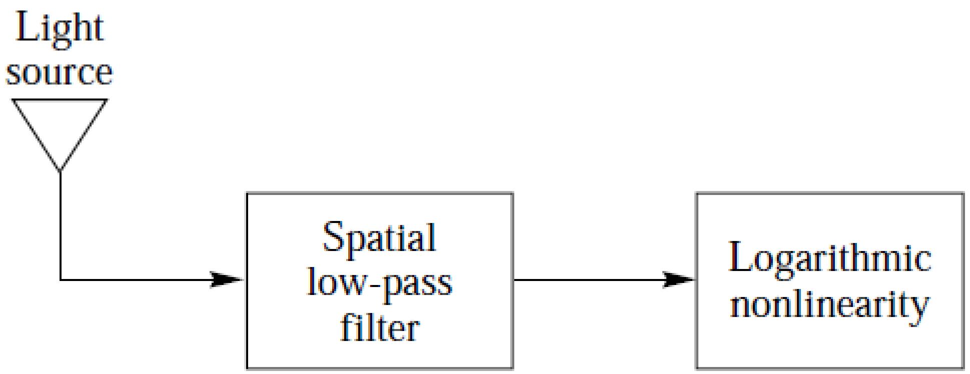
Human Visual System (HVS) (4)



- The eye acts as a spatial low-pass filter.
- We can model the eye as a receptor whose output goes to a logarithmic nonlinearity.
- *The mind does not perceive everything that the eye sees!*
- We can use this knowledge to design compression systems such that the distortion introduced by our lossy compression scheme is not noticeable.

Lossy Compression

Human Auditory Perception

- The Human Ear
  - Divided into three parts:
    - The outer ear:
      - the structure that directs that sound waves, or pressure waves, to the tympanic membrane, or eardrum.
      - This membrane separates the outer ear from the middle ear.
    - The middle ear
      - An air-filled cavity containing three small bones that provide coupling between the tympanic membrane and the oval window, which leads into the inner ear.
      - The tympanic membrane and the bones convert the pressure waves in the air to acoustical vibrations.
    - The inner ear
      - Contains, among other things, a snail-shaped passage called the cochlea that contains the transducers that convert the acoustical vibrations to nerve impulses.

Lossy Compression

Human Auditory Perception

- The Human Ear (2)
  - Can hear sounds in the frequency range 20 Hz to 20 kHz, a 1000:1 range of frequencies.
    - 25 overlapping critical bands
  - Bad news!
    - This range decreases with old age
    - Older people are usually unable to hear the higher frequencies.

Lossy Compression

Rate-Distortion Theory

- Concerned with the trade-offs between distortion and rate in lossy compression schemes.
- Rate is defined as the average number of bits used to represent each sample value.
- One way of representing the trade-offs is via a rate distortion function R(D).
- R(D) specifies the lowest rate at which the output of a source can be encoded while keeping the distortion less than or equal to D.

$$D = E[d(X,Y)] = \sum_{i=0}^{N-1} \sum_{j=0}^{M-1} d(x_i, y_j)\, P(x_i, y_j)$$

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Rate-Distortion Theory (2)

- The minimum rate for a given distortion is given by

$$R(D) = \min_{\{P(y_j|x_i)\}\in\Gamma} I(X;Y)$$

Where $\Gamma = \left\{\{P\left(y_j\middle|x_i\right)\} such\ that\ D\left(\{P\left(y_j\middle|x_i\right)\}\right) \leq D^*\right\}$ is determined by the compression scheme

- $H(Y|X) = 0 \rightarrow I(X;Y) = H(Y)$
- $H(Y|X) = H(Y) \rightarrow I(X;Y) = 0$

Lossy Compression

Models

- If the sources can be modelled accurately, we would be able to derive more accurate RD relationships for coding decisions
- Popular models
  - Probability models
  - Linear system models

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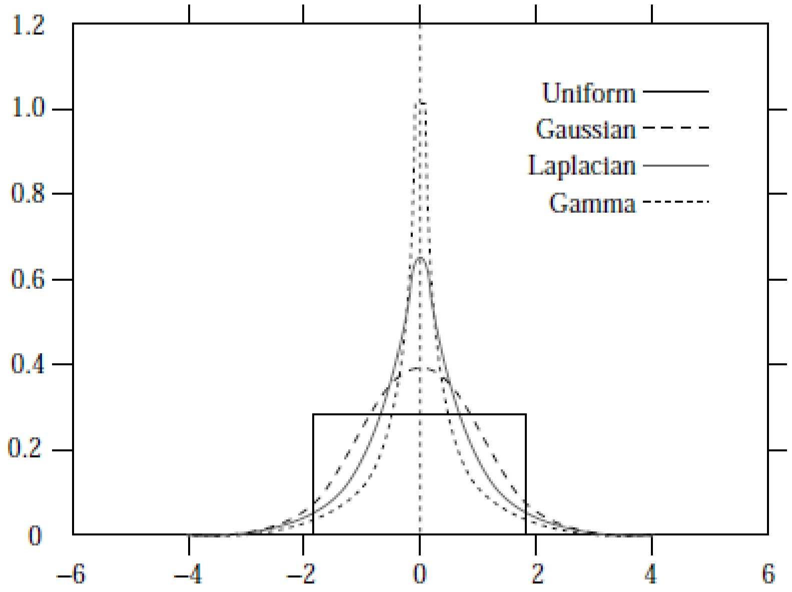
Models

- Probability models
  - Uniform distribution
  - Gaussian distribution
  - Laplacian distribution
  - Gamma distribution



Lossy Compression

- Autoregressive Moving Average Model: ARMA(N, M)

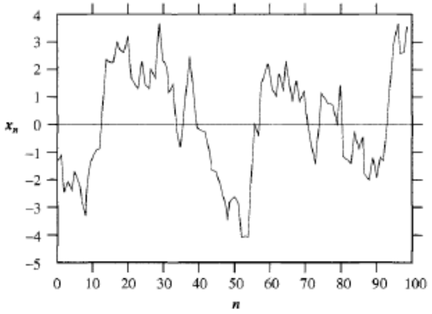
$$x_n = \sum_{i=1}^{N} a_i x_{n-i} + \sum_{j=1}^{M} b_i \varepsilon_{n-j} + \varepsilon_n.$$

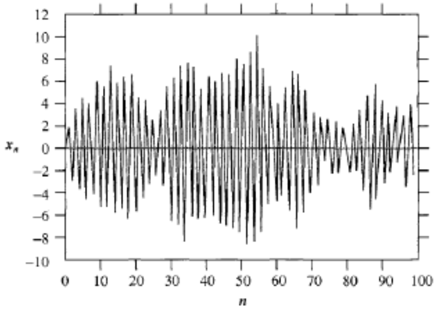
- Autoregressive Model: AR(N)

$$x_n = \sum_{i=1}^{N} a_i x_{n-i} + \varepsilon_n.$$

  - AR(N) is a Markov Model of order N.

- Examples of AR(1) sources:





Lossy Compression

Auto Correlation Function

- The autocorrelation function for the AR(N) process can be obtained as follows:

$$R_{xx}(k) = E[x_n x_{n-k}] = E\left[\left(\sum_{i=1}^{N} a_i x_{n-i} + \varepsilon_n\right) x_{n-k}\right]$$

$$= E\left[\sum_{i=1}^{N} a_i x_{n-i} x_{n-k}\right] + E[\varepsilon_n x_{n-k}] = \begin{cases} \sum_{i=1}^{N} a_i R_{xx}(k-i), & k > 0 \\ \sum_{i=1}^{N} a_i R_{xx}(i) + \sigma_\varepsilon^2, & k = 0 \end{cases}$$

- Autocorrelation function of a process tells us the sample-to-sample behavior of a sequence
  - Slowly decay w.r.t. $k \rightarrow$ high sample-to-sample correlation
  - Fast decay w.r.t. $k \rightarrow$ low sample-to-sample correlation
  - No sample-to-sample correlation $\rightarrow$ zero (except when $k = 0$).

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