EEE 324 Digital Signal Processing

Lecture 20

Linear Systems with Generalized Linear Phase

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- Causal Generalized Linear Phase Systems
- Locations of Zeros for FIR Linear Phase Systems
- Relation of FIR Linear Phase Systems to Minimum Phase Systems

Linear Systems with Generalized Linear Phase

Causal GLP Systems

- Causal FIR systems have generalized linear phase if they have IR length (M+1) and satisfy either Eq. (5) or Eq. (6)

If

$$h[n] = \begin{cases} h[M-n], & 0 \le n \le M \\ 0, & otherwise \end{cases} \qquad (7)$$

then

$$H(e^{j\omega}) = A_e(e^{j\omega})e^{-j\omega M/2}$$

where

$A_e(e^{j\omega})$ is an even function of ω

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If

$$h[n] = \begin{cases} -h[M-n], & 0 \le n \le M \\ 0, & otherwise \end{cases} \quad (8)$$

then

$$H(e^{j\omega}) = jA_o(e^{j\omega})e^{-j\omega M/2} = A_o(e^{j\omega})e^{-\frac{j\omega M}{2}+\frac{j\pi}{2}}$$

where

$A_o(e^{j\omega})$ is an odd function of ω

- In both the cases, the length of the IR is $(M+1)$ samples.
- Eq. (7) and Eq. (8), are sufficient to guarantee a causal system with GLP.

Linear Systems with Generalized Linear Phase

Causal GLP Systems

Based on the (even/odd) nature of M and the (even/odd) symmetry of type of symmetry of the IR, four different types of FIR LPS have been defined:

1. TYPE I
2. TYPE II
3. TYPE III
4. TYPE IV

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TYPE I

IR: Even Symmetric M: Even Example (M=2): h[n]={1,2,1}

$$h[n] = h[M-n], \qquad 0 \le n \le M$$

Since M is an even integer, the delay M/2 is an integer.

The FR is

$$H(e^{j\omega}) = \sum_{n=0}^{M} h[n]e^{-j\omega n}$$

$$H(e^{j\omega}) = e^{-\frac{j\omega M}{2}}\left(\sum_{k=0}^{\frac{M}{2}} a[k]cos\omega k\right)$$

Where

$$a[0] = h\left[\frac{M}{2}\right]$$

$$a[k] = 2h\left[\left(\frac{M}{2}\right) - k\right], \quad k = 1,2,\dots,\frac{M}{2}$$

β in Eq. (3) is either 0 or π

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TYPE II

IR: Even Symmetric ***M: Odd*** ***Example (M=3): h[n]= {1,2,2,1}***

$$h[n] = h[M-n], 0 \le n \le M$$

$$H(e^{j\omega}) = e^{-\frac{j\omega M}{2}}\left(\sum_{k=1}^{\frac{M+1}{2}} b[k]\cos\left[\omega\left(k-\frac{1}{2}\right)\right]\right)$$

$$b[k] = 2h\left[\frac{(M+1)}{2}-k\right], \quad k = 1,2,\ldots,\frac{(M+1)}{2}$$

β in Eq. (3) is either 0 or π

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TYPE III

IR: Odd Symmetric ***M: Even*** ***Example (M=2): h[n]={1,0,-1}***

$$h[n] = -h[M-n], \qquad 0 \le n \le M$$

$$H(e^{j\omega}) = je^{-\frac{j\omega M}{2}} \left(\sum_{k=1}^{\frac{M}{2}} c[k] \sin \omega k \right)$$

$$c[k] = 2h\left[\frac{M}{2} - k\right], \quad k = 1,2, \dots, \frac{M}{2}$$

β in Eq. (3) is either $\frac{\pi}{2}$ or $\frac{3\pi}{2}$

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TYPE IV

IR: Odd Symmetric M: Odd ***Example (M=3): h[n]={1,-2,2,-1}***

$$h[n] = -h[M-n], \qquad 0 \leq n \leq M$$

$$H(e^{j\omega}) = je^{-\frac{j\omega M}{2}} \left(\sum_{k=1}^{\frac{(M+1)}{2}} d[k] \sin\left[\omega\left(k - \frac{1}{2}\right)\right] \right)$$

$$d[k] = 2h\left[\frac{(M+1)}{2} - k\right], \quad k = 1,2,\dots,\frac{(M+1)}{2}$$

β in Eq. (3) is either $\frac{\pi}{2}$ or $\frac{3\pi}{2}$

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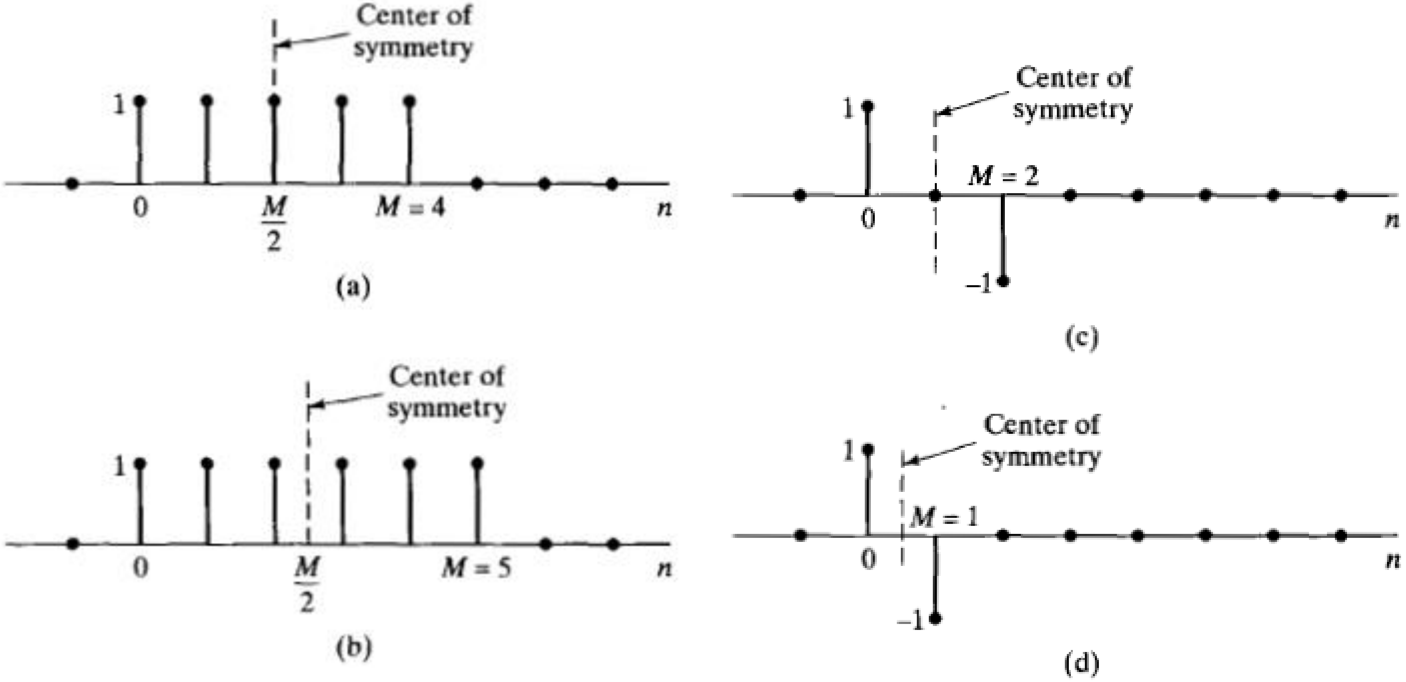

Figure 5.36 Examples of FIR linear-phase systems. (a) Type I, M even, $h[n] = h[M-n]$. (b) Type II, M odd, $h[n] = h[M-n]$. (c) Type III, M even, $h[n] = -h[M-n]$. (d) Type IV, M odd, $h[n] = -h[M-n]$.

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Causal GLP Systems

Type	Impulse response symmetry	Impulse response length
I	symmetric	$M+1$ odd (order is even)
II	symmetric	$M+1$ even (order is odd)
III	antisymmetric	$M+1$ odd (order is even)
IV	antisymmetric	$M+1$ even (order is odd)

Type	LPF	HPF	BPF	BSF	Comment
I	Y	Y	Y	Y	Most versatile.
II	Y	N	Y	N	Zero at $z=-1$.
III	N	N	Y	N	Zeros at $z=\pm 1$.
IV	N	Y	Y	N	Zero at $z=1$.

Ref: Dr. Richard Brown III (Lecture Notes)

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