EEE 324 Digital Signal Processing

Lecture 21

Location of Zeros for FIR Linear Phase Systems

Dr. Shadan Khattak

Department of Electrical Engineering

COMSATS Institute of Information Technology - Abbottabad

Contents

- Locations of Zeros for FIR Linear Phase Systems
- Relation of FIR Linear Phase Systems to Minimum Phase Systems

Dr. Shadan Khattak
Department of Electrical Engineering
CIIT - Abbottabad

Locations of Zeros for FIR Linear Phase Systems

In this lecture, we will consider the locations of the zeros of the system function for FIR linear phase systems.

The system function is

$$H(z) = \sum_{n=0}^{M} h[n] z^{-n}$$

Locations of Zeros for FIR Linear Phase Systems

<u>In case of even symmetry (Type I & Type II)</u>

$$H(z) = \sum_{n=0}^{M} h[M-n]z^{-n}$$

$$= \sum_{k=0}^{M} h[k]z^{k}z^{-M}$$

$$= z^{-M}H(z^{-1}) \qquad (1)$$

From Eq. (1), we conclude that if z_0 is a zero of $H(z)$, then

$$H(z_0) = {z_0}^{-M}H({z_0}^{-1}) = 0$$

If

$$z_0 = re^{j\theta} \text{ is a zero of } H(z)$$

Then

$$z_0^{-1} = r^{-1}e^{-j\theta} \text{ is also a zero of } (h)$$

Locations of Zeros for FIR Linear Phase Systems

In case of even symmetry (Type I & Type II)

- When $h[n]$ is real, each complex zero not on the unit circle will be part of a set of four conjugate reciprocal zeros of the form

$$\left(1-re^{j\theta}z^{-1}\right)\left(1-re^{-j\theta}z^{-1}\right)\left(1-r^{-1}e^{j\theta}z^{-1}\right)\left(1-r^{-1}e^{j\theta}z^{-1}\right)$$

  - Zeros on the unit circle (i.e., $r=1$) come in pairs of the form
  $$\left(1-e^{j\theta}z^{-1}\right)\left(1-e^{-j\theta}z^{-1}\right)$$
  - If a zero is real and not on the unit circle, the reciprocal will also be a zero of $H(z)$ and $H(z)$ will have factors of the form
  $$\left(1\pm re^{j\theta}z^{-1}\right)\left(1\pm r^{-1}e^{-j\theta}z^{-1}\right)$$
  - If zero is at $z=\pm1$, it can appear all by itself, since it will be its own conjugate. So then $H(z)$ will be of the form
  $$(1\pm z^{-1})$$

Locations of Zeros for FIR Linear Phase Systems

In case of even symmetry (Type I & Type II)

- When $z = -1$

$$\Rightarrow H(-1) = (-1)^M H(-1)$$

  - If M is even (Type I FIR), $H(-1)$ can be of any value.
  - If M is odd (Type II FIR), $H(-1) = 0$ is the only possible value for $z = -1$.

So for symmetric IR with M odd (Type II FIR), the system function must have a zero at $z = -1$.

- For the locations of the compulsory zeros of Type III and Type IV systems, and for the implication of the locations of zeros, please see the end slides of the previous lecture.

Locations of Zeros for FIR Linear Phase Systems

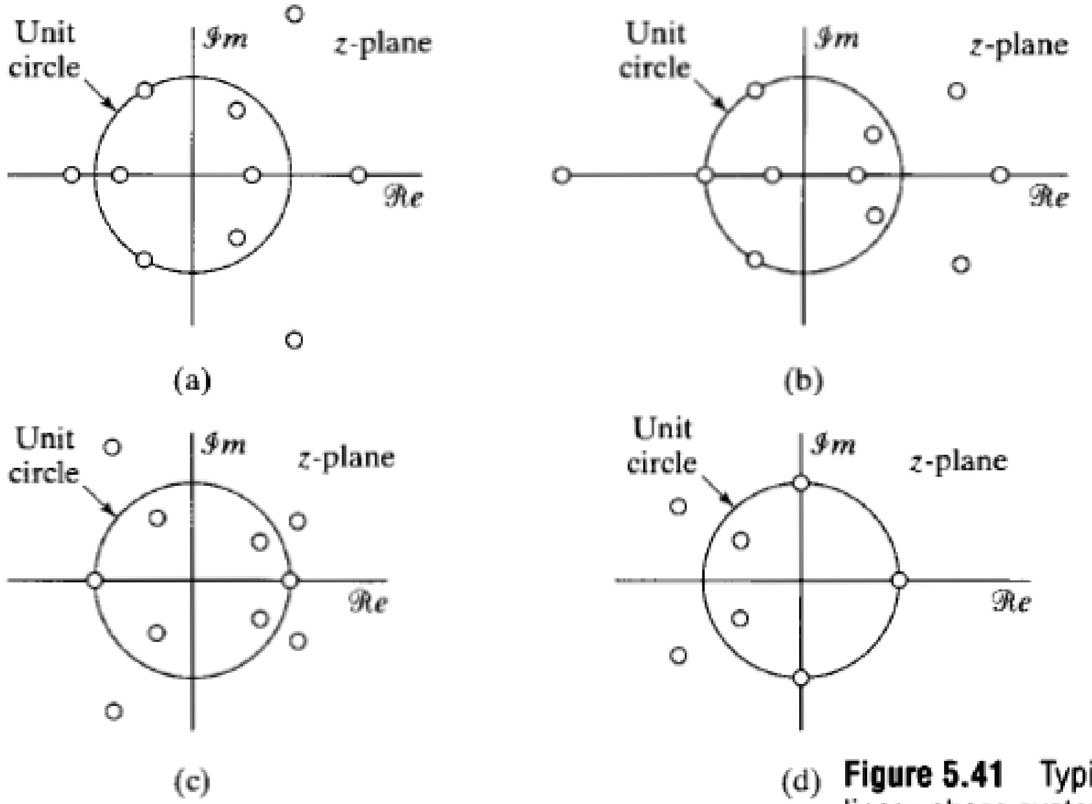

Figure 5.41 Typical plots of zeros for linear-phase systems. (a) Type I. (b) Type II. (c) Type III. (d) Type IV.

Dr. Shadan Khattak
Department of Electrical Engineering
CIIT - Abbottabad

Practice Problems

<u>Exercise Problems: 5.1 – 5.29</u>