



COMSATS Institute of
Information Technology

EEE 324 Digital Signal Processing

Lecture 22

Introduction to Filter Design Techniques

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- Introduction to Filter Design Techniques

Introduction

- Filters are an important class of LTI systems.
- In this chapter, we will concentrate on the design of causal filters.
 - Very often, non-causal filters are designed and implemented by modifying causal filters.

Introduction

Different Stages of Filter Design

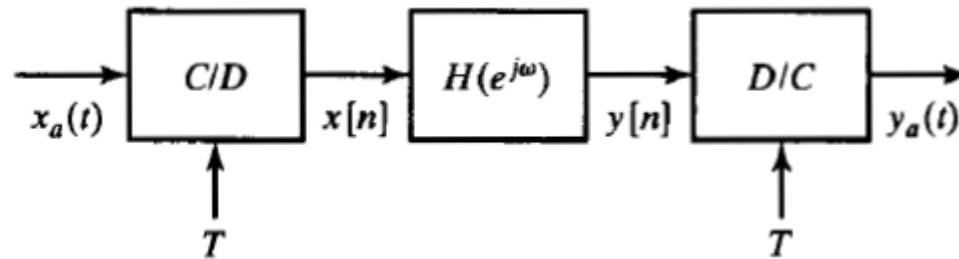
1. Specification of the desired properties of the system.
 - Depends on the application.
2. The approximation of the specifications using a causal DT system.
 - We will mainly focus on this stage in this chapter.
3. The realization of the system.
 - Depends on the technology to be used for the implementation.

Introduction

- *Practically, a desired filter is generally implemented with digital computation and used to filter a signal that is derived from a CT signal with the help of periodic sampling followed by A/D conversion.*
 - *That is why DT filters are often referred to as digital filters.*

Introduction

- When a DT filter is to be used for DT processing of CT signals in the configuration of the following figure, the specs for both the DT filter and the effective CT filter are typically given in the frequency domain.



Introduction

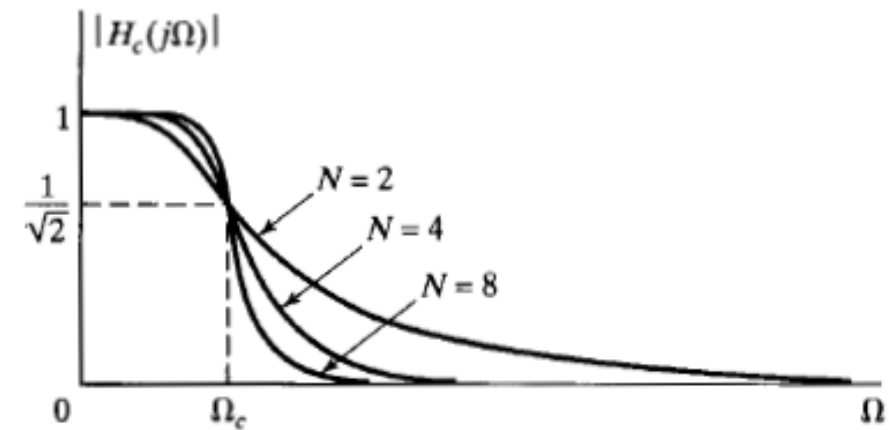
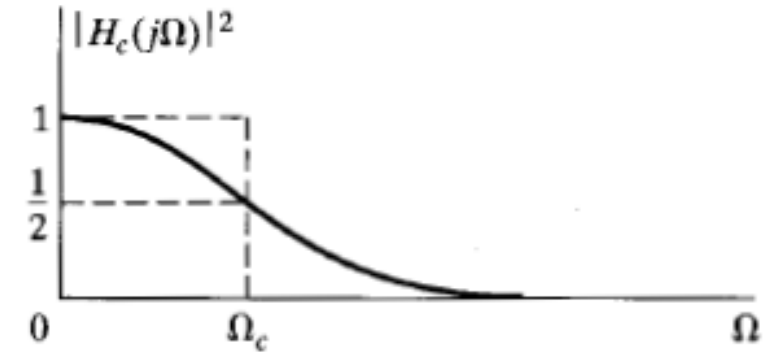
Example 7.1

Butterworth Lowpass Filters

- The MR is maximally flat in the PB.
- The MR is monotonic in the PB and the SB.
- The MSR for a CT Butterworth lowpass filter is of the form

$$|H_c(j\Omega)|^2 = \frac{1}{1+(j\Omega/j\Omega_c)^{2N}} \quad (1)$$

- As the order N increases, the filter characteristics become sharper.
 - Remain close to unity over more of the PB
 - Become close to zero more rapidly in the SB.



Butterworth Lowpass Filters

- In Eq. (1), substituting $j\Omega$ with s ,

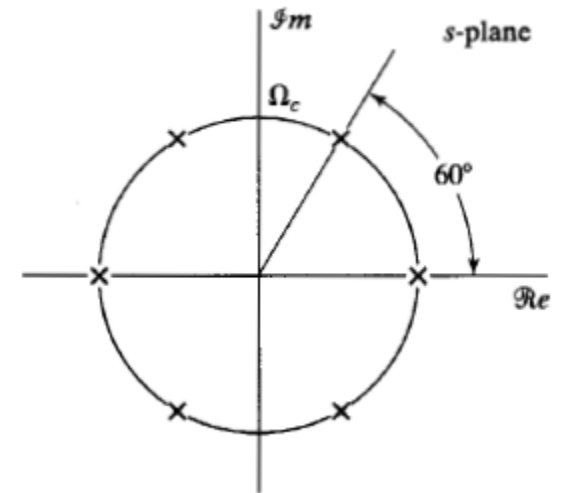
$$H_c(s)H_c(-s) = \frac{1}{1 + (s/j\Omega_c)^{2N}}$$

- The roots of the denominator are:

$$s_k = (-1)^{\frac{1}{2N}}(j\Omega_c) = \Omega_c e^{(j\pi/2N)(2k+N-1)}$$

Butterworth Lowpass Filters

- There are $2N$ poles equally spaced in angle on a circle of radius Ω_c in the $s - plane$.
- The poles are symmetrically placed with respect to the imaginary axis.
- A pole never falls on the imaginary axis.
- One pole occurs on the real axis for N odd.
- No pole occurs on the real axis for N even.
- The angular spacing between the poles on the circle is π/N radians.



Butterworth Lowpass Filters

- The poles of the MSF always occur in pairs i.e.,
 - If there is a pole at $s = s_k$, then a pole also occurs at $s = -s_k$.
 - So, to construct $H_c(s)$ from the MSF, we should choose one pole from each pair.
- To obtain a stable and causal filter, we should choose the poles on the left half plane part of the s-plane.