EEE 324 Digital Signal Processing

Lecture 3

Reconstruction of a Band-limited Signal from its Samples

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Contents

- Reconstruction of a band-limited (BL) signal from its samples.

Reconstructing $x_c(t)$ from samples $x[n]$

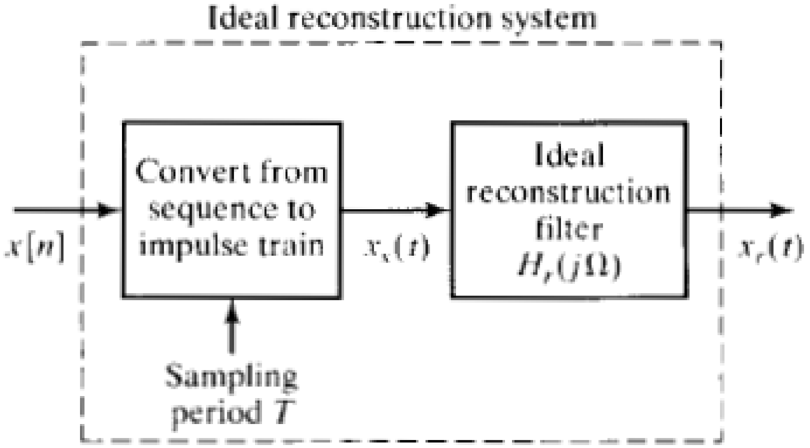
- Given, x[n], first, form an impulse train $x_s(t)$ in which successive impulses are assigned an area equal to successive sequence values i.e.,

$$x_s(t) = \sum_{n=-\infty}^{\infty} x[n]\delta(t - nT)$$

$$x_r(t) = x_s(t) * h_r(t)$$

$$x_r(t) = \sum_{n=-\infty}^{\infty} x[n]\delta(t - nT) * h_r(t)$$

$$\boxed{x_r(t) = \sum_{n=-\infty}^{\infty} x[n]h_r(t - nT)} \quad (11)$$

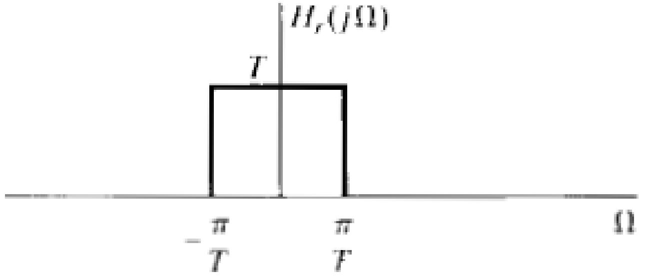


- We need to define $h_r(t)$.
- We will do this in the next couple of slides.

Reconstructing $x_c(t)$ from samples $x[n]$

The Ideal Reconstruction Filter $H_r(j\Omega)$

- Gain of T
- Cut-off frequency Ω_c *between* Ω_N *and* $\Omega_s - \Omega_N$
- A convenient choice of Cut-off frequency $\Omega_c = \frac{\Omega_s}{2} = \frac{\pi}{T}$

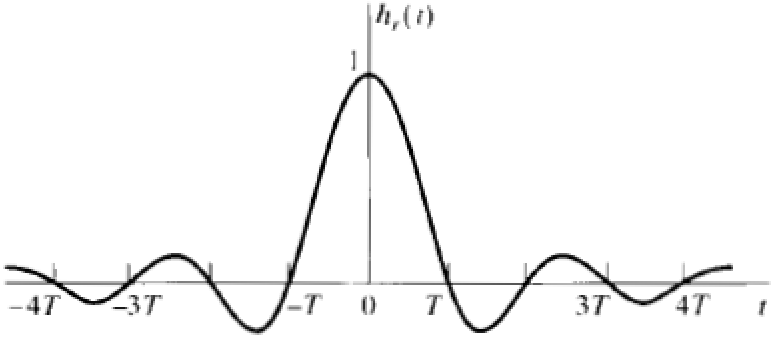


- We know that,

$$\frac{sinWt}{\pi t} \overset{CTFT}{\longleftrightarrow} \begin{cases} 1, & |\omega| < W \\ 0, & |\omega| > W \end{cases}$$

- Here, $W = \frac{\pi}{T}$
- In Eq. (11), $h_r(t)$ is the inverse CTFT of $H_r(j\Omega)$.
- So,

$$\boxed{h_r(t) = \frac{\sin\left(\frac{\pi t}{T}\right)}{\frac{\pi t}{T}}} \quad (12)$$



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Reconstructing $x_c(t)$ from samples $x[n]$

The Reconstructed Signal $x_r(t)$

The reconstructed signal $x_r(t)$ can be written in terms of the samples $x[n]$ and the impulse response of the ideal reconstruction filter $h_r(t)$.

Putting Eq. (12) into Eq. (11),

$$x_r(t) = \sum_{n=-\infty}^{\infty} x[n] \frac{\sin[\pi\left(\frac{t-nT}{T}\right)]}{\frac{\pi(t-nT)}{T}} \qquad (13)$$

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Reconstructing $x_c(t)$ from samples $x[n]$

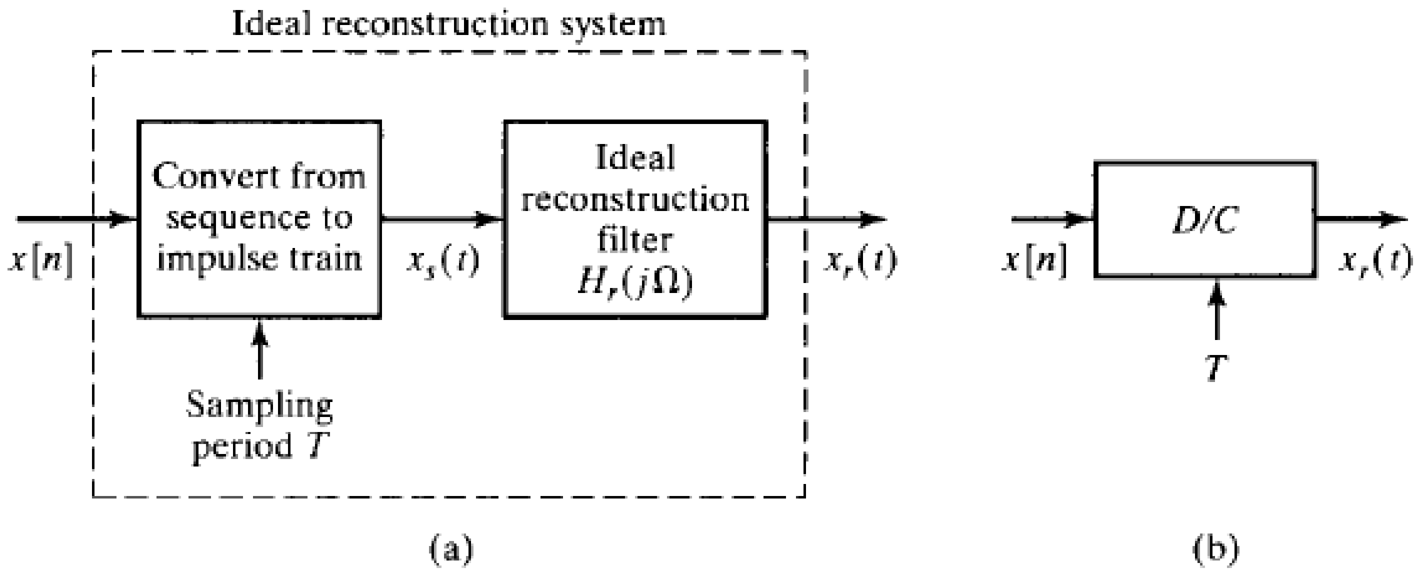

Figure 4.10 (a) Ideal bandlimited signal reconstruction. (b) Equivalent representation as an ideal D/C converter.

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Reconstructing $x_c(t)$ from samples $x[n]$

Frequency domain representation

- In the previous lecture, we saw that

if,	$x[n] = x_c(nT)$
where,	$X_c(j\Omega) = 0$ for $\vert\Omega\vert \geq \pi/T$
then,	$x_r(t) = x_c(t)$

Reconstructing $x_c(t)$ from samples $x[n]$

Frequency domain representation

Consider the CTFT of Eq. (11)

$$X_r(j\Omega) = \int_{-\infty}^{\infty} x_r(t)e^{-j\Omega t}dt$$

Putting the value of $x_r(t)$ from Eq. (11)

$$X_r(j\Omega) = \int_{t=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} x[n]h_r(t-nT)\, e^{-j\Omega t}dt$$

$$X_r(j\Omega) = \sum_{n=-\infty}^{\infty} x[n] \int_{t=-\infty}^{\infty} h_r(t-nT)e^{-j\Omega t}\, dt$$

Using the time shift property of CTFT,

$$\int_{t=-\infty}^{\infty} h_r(t-nT)e^{-j\Omega t}\, dt = H_r(j\Omega)e^{-j\Omega nT}$$

So,

$$X_r(j\Omega) = \sum_{n=-\infty}^{\infty} x[n]H_r(j\Omega)e^{-j\Omega Tn}$$

$$\boxed{X_r(j\Omega) = H_r(j\Omega)X(e^{j\Omega T})} \qquad (14)$$

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Reconstructing $x_c(t)$ from samples $x[n]$

Frequency domain representation

- $X(e^{j\omega})$ is frequency scaled (i.e. ω is replaced by ΩT).
- The output of the ideal D/C converter is always band-limited to at most the cut-off frequency of the LP filter, which is usually chosen as one-half the sampling frequency.

Reading

Section 4.3 (Oppenheim)

Practice Problems

Problem 4.24 (Oppenheim)